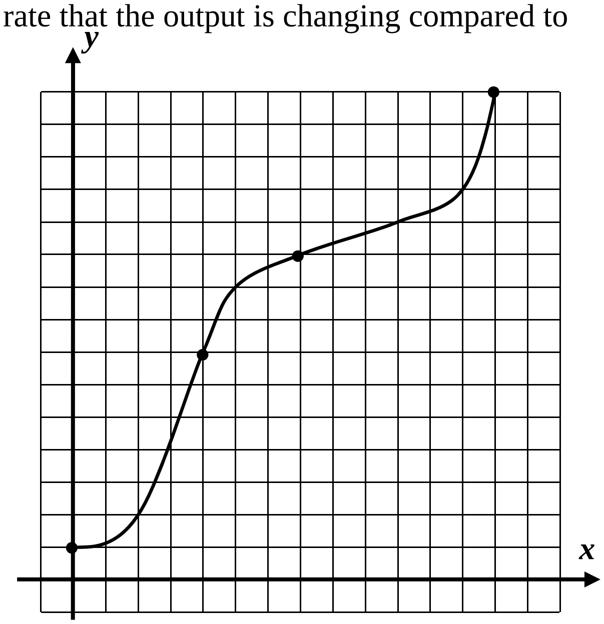
Name: ______________________________ Date: ________________

A2CC Average Rate of Change

Warm Up:
Determine the equation of the parabola whose focus is (0,8) and whose directrix is the horizontal line y = 2?

When we model using functions, we are very often interested in the rate that the output is changing compared to the rate of the input.



***Exercise* #1:** The function $f(x)$ is shown graphed to the right.

(a) Evaluate each of the following based on the graph:

(i) $f(0)$ (ii) $f(4)$ (iii) $f(7)$ (iv) $f(13)$

(b) Find the change in the function, Δf, over each of the following domain intervals. Find this both by subtraction and show this on the graph.

(i) $0 \le x \le 4$ (ii) $4 \le x \le 7$ (iii) $7 \le x \le 13$

(c) Why can't you simply compare the changes in *f* from part (b) to determine over which interval the function changing the fastest?

(d) Calculate the **average rate of change** for the function over each of the intervals and determine which interval has the greatest rate.

(i) $0 \le x \le 4$ (ii) $4 \le x \le 7$ (iii) $7 \le x \le 13$

(e) Using a straightedge, draw in the lines whose slopes you found in part (d) by connecting the points shown on the graph. The average rate of change gives a measurement of what property of the line?

The average rate of change is an exceptionally important concept in mathematics because it gives us a way to **quantify** how fast a function changes on average over a certain **domain interval**. Although we used its formula in the last exercise, we state it formally here:

AVERAGE RATE OF CHANGE

For a function over the domain interval $a \le x \le b$, the function's **average rate of change** is calculated by:

$$\frac{\Delta f}{\Delta x} = \frac{\text{change in the output}}{\text{change in the input}} = \frac{f(b) - f(a)}{b - a}$$

***Exercise* #2:** Consider the two functions $f(x)=5x+7$ and $g(x)=2x^2+1$.

(a) Calculate the average rate of change for both functions over the following intervals. Do your work carefully and show the calculations that lead to your answers.

(i) $-2 \le x \le 3$

(ii) $1 \le x \le 5$

(b) The average rate of change for *f* was the same for both (i) and (ii) but was not the same for *g*. Why is that?

***Exercise* #3:** The table below represents a linear function. Fill in the missing entries.

x	1	5	11		45
y	-5	1		22	

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AVERAGE RATE OF CHANGE HOMEWORK

1. For the function $g(x)$ given in the table below, calculate the average rate of change for each of the following intervals.

x	-3	-1	4	6	9
$g(x)$	8	-2	13	12	5

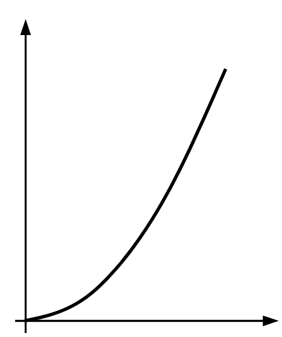
(a) $-3 \le x \le -1$ (b) $-1 \le x \le 6$ (c) $-3 \le x \le 9$

(d) Explain how you can tell from the answers in (a) through (c) that this is **not** a table that represents a linear function.

2. Consider the simple quadratic function $f(x) = x^2$. Calculate the average rate of change of this function over the following intervals:

(a) $0 \le x \le 2$ (b) $2 \le x \le 4$ (c) $4 \le x \le 6$

(d) Clearly the average rate of change is getting larger at x gets larger. How is this reflected in the graph of f shown sketched to the right?



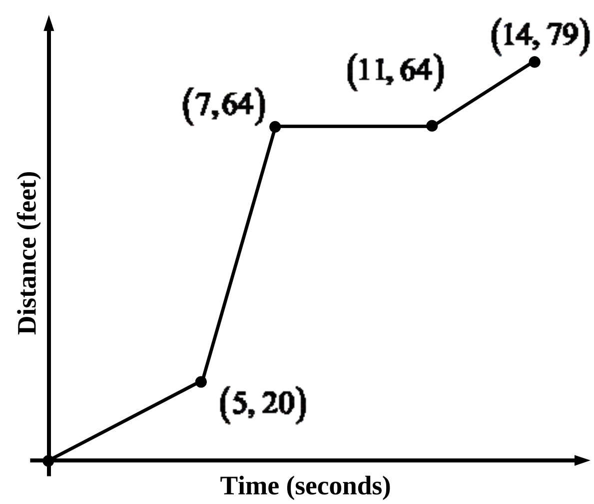
3. Which has a greater average rate of change over the interval $-2 \le x \le 4$, the function $g(x) = 16x - 3$ or the function $f(x) = 2x^2$? Provide justification for your answer.

APPLICATIONS

4. An object travels such that its distance, *d*, away from its starting point is shown as a function of time, *t*, in seconds, in the graph below.

   (a) What is the average rate of change of *d* over the interval $5 \le t \le 7$? Include proper units in your answer.

   (b) The average rate of change of distance over time (what you found in part (a)) is known as the **average speed** of an object. Is the average speed of this object greater on the interval $0 \le t \le 5$ or $11 \le t \le 14$? Justify.



REASONING

5. What makes the average rate of change of a linear function different from that of any other function? What is the special name that we give to the average rate of change of a linear function?